NWC 2017 NAFEMS WORLD CONGRESS

11-14 JUNE | STOCKHOLM | SWEDEN

A WORLD OF ENGINEERING SIMULATION

GDTECH engineering

Optimisation of variable thickness composite structures with continuous design variables

M. Bruyneel, GDTech & University of Liège, Belgium
P. Duysinx, University of Liège, Belgium
S. Grihon, Airbus Toulouse, France

Outline of the presentation

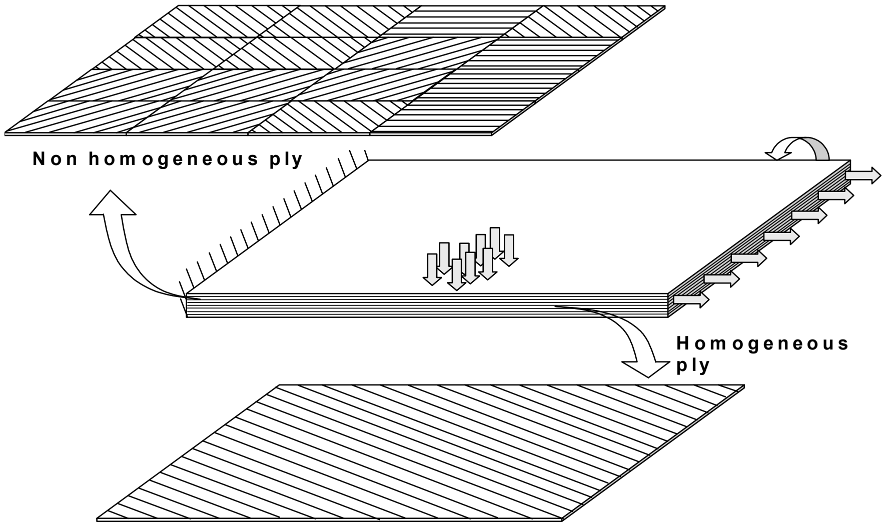
- Problem statement
- Material parameterization
- Optimization algorithm
- Design rules constraints
- Ply continuity constraint
- Generalization
- Conclusions

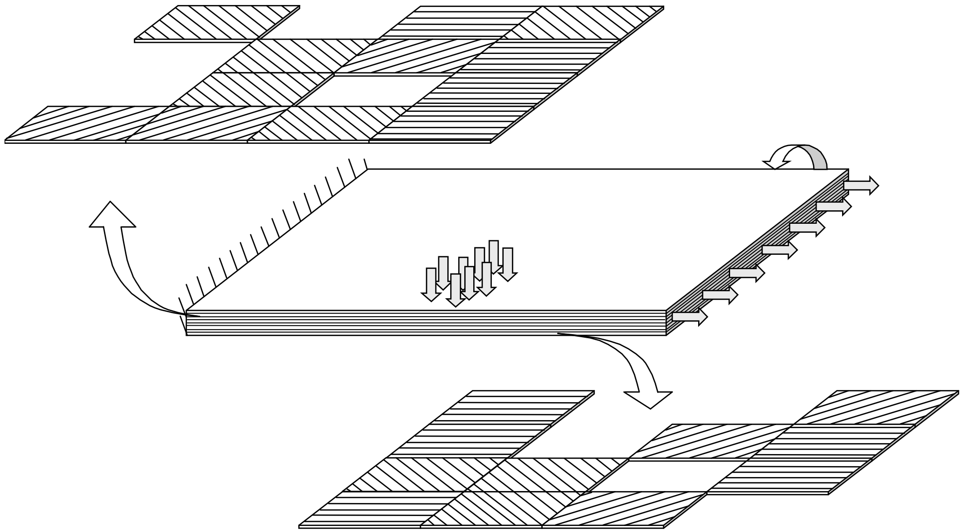
Outline of the presentation

- Problem statement
- Material parameterization
- Optimization algorithm
- Design rules constraints
- Ply continuity constraint
- Generalization
- Conclusions

Problem statement

- Selection of fibers orientation
  - Which fiber orientation should I define in ply *k* of region *r*?
  - Knowing that conventional orientation (0°, -45°, 45°, 90°) must be used
  - Knowing that design rules must be taken into account
  - Knowing that manufacturing constraint (ply drop/ply continuity) exists



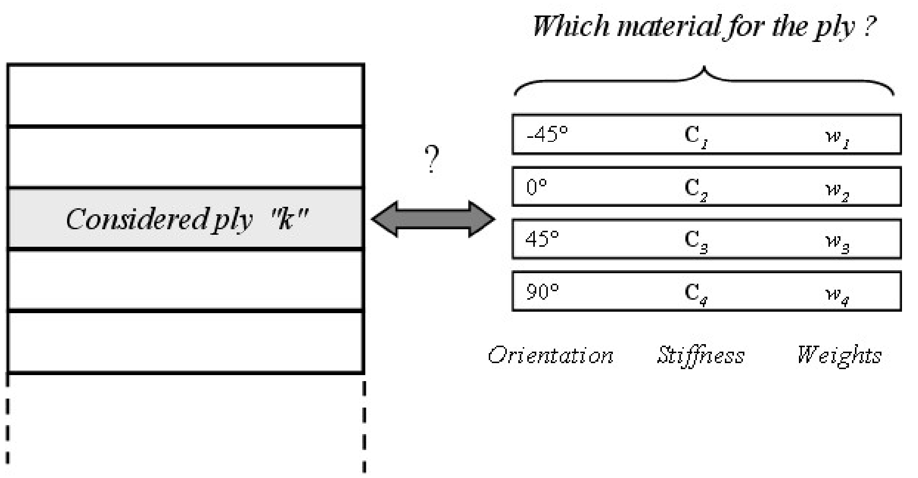


Outline of the presentation

- Problem statement
- Material parameterization
- Optimization algorithm
- Design rules constraints
- Ply continuity constraint
- Generalization
- Conclusions

Material parameterization

- Conventional orientations are used: -45°, 0°, 45°, 90°
  - By nature, discrete design variables
  - Here, the problem is transformed to play with **continuous design variables**
    - – Specific parameterization of the material stiffness matrix
    - – Here, 4 candidate materials (0°,45°,-45°,90°)



$$\boldsymbol{\sigma}^k = \mathbf{C}^k \boldsymbol{\varepsilon}^k$$

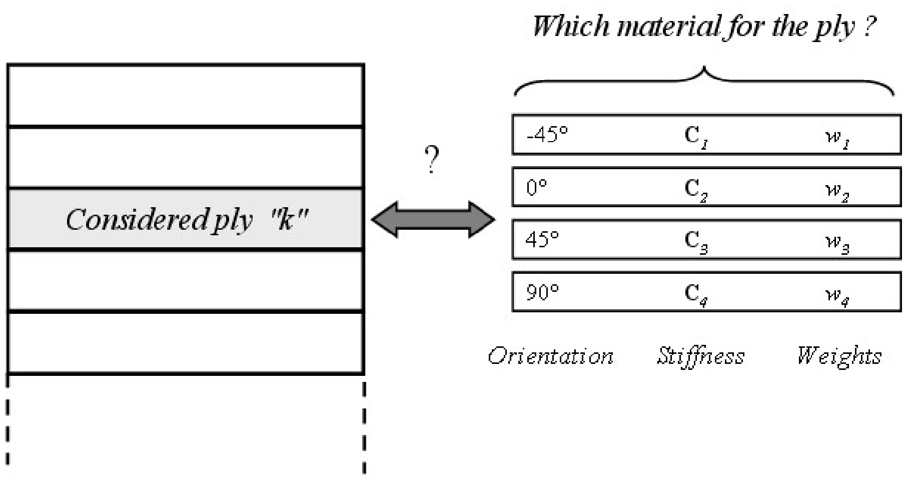
$$\mathbf{C}^k = \sum_{i=1}^{4} w_i^k \mathbf{C}_i^k$$

with
$$\mathbf{C}_1^k = \mathbf{C}_{-45°}$$
$$\mathbf{C}_2^k = \mathbf{C}_{0°}$$
$$\mathbf{C}_3^k = \mathbf{C}_{45°}$$
$$\mathbf{C}_4^k = \mathbf{C}_{90°}$$

Material parameterization

- Conventional orientations are used: -45°, 0°, 45°, 90°
  - By nature, discrete design variables
  - Here, the problem is transformed to play with **continuous design variables**
    - – Specific parameterization of the material stiffness matrix
    - – Here, 4 candidate materials (0°,45°,-45°,90°)

$$\boldsymbol{\sigma}^k = \mathbf{C}^k \boldsymbol{\varepsilon}^k$$



$$\mathbf{C}^k = w_1^k \mathbf{C}_{-45}^k + w_2^k \mathbf{C}_0^k + w_3^k \mathbf{C}_{45}^k + w_4^k \mathbf{C}_{90}^k$$

$$\sum_{i=1}^{n^k} w_i^k = 1$$

$$0 \le w_i^k \le 1 \qquad i = 1,\ldots,n^k$$

Material parameterization

– Key issue: definition of the weighting functions w_i in $\mathbf{C}^k = \sum_{i=1}^{n^k} w_i^k \mathbf{C}_i^k$

– In the literature:

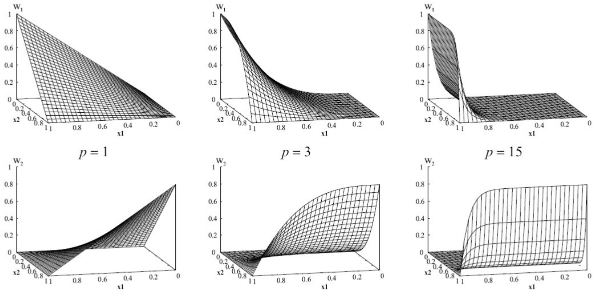
- DMO (Discrete Material Optimization) by Lund & co-workers (from 2005)
- n design variables if n candidate materials

DMO4

$$w_i^{\mathrm{DMO4}} = (x_i)^p \prod_{j=1;\, j\neq i}^{n} \left[1-(x_j)^p\right]$$

x_i => continuous design variables $\in \,]0;1]$

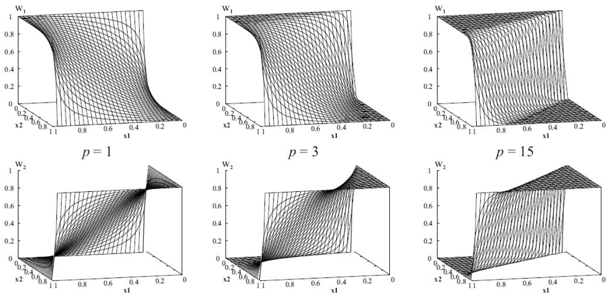
Ex. $w_1 = x_1(1-x_2^p)(1-x_3^p)(1-x_4^p)$



DMO5

$$w_i^{\mathrm{DMO5}} = \frac{w_i^{DMO4}}{\sum_{k=1}^{n^l} w_k^{DMO4}}$$

Scaled version of DMO4



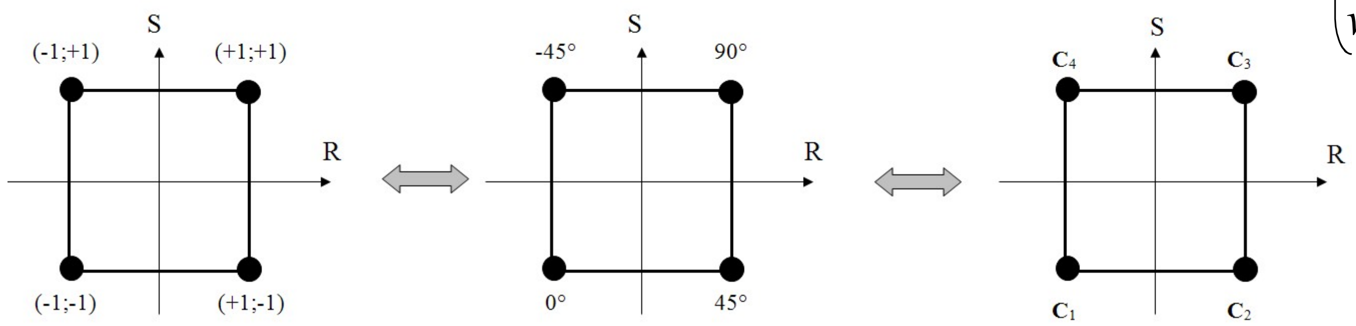
Material parameterization

- Key issue: definition of the weighting functions w_i in $\mathbf{C}^k = \sum_{i=1}^{n^k} w_i^k \mathbf{C}_i^k$
- In the literature:
  - SFP (Shape Function Parameterization) by Bruyneel (2011)
    - Here for 4 candidate materials, but possible extension to more (or less) materials
    - 2 design variables for 4 candidate materials

$$\sum_{i=1}^{4} w_i^k = 1$$

=> Use of the finite elements shape functions for a quadrangular element

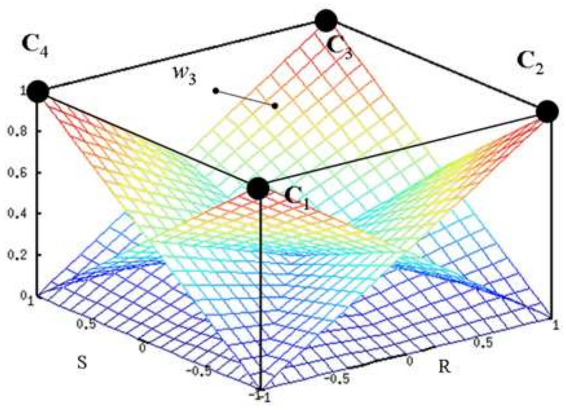
$$\begin{cases} w_1 = 0.25(1-R)(1-S) \\ w_2 = 0.25(1+R)(1-S) \\ w_3 = 0.25(1+R)(1+S) \\ w_4 = 0.25(1-R)(1+S) \end{cases}$$



Material parameterization

- Key issue: definition of the weighting functions w_i in $\mathbf{C}^k = \sum_{i=1}^{n^k} w_i^k \mathbf{C}_i^k$
- In the literature:
  - SFP (Shape Function Parameterization) by Bruyneel (2011)
    - Here for 4 candidate materials, but possible extension to more (or less) materials
    - 2 design variables for 4 candidate materials

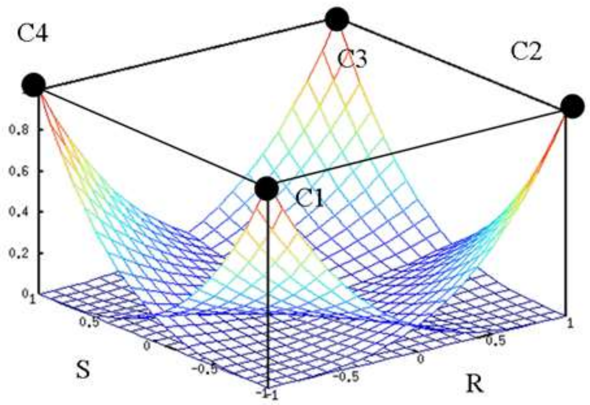
SF: Shape Functions



$$w_i^{SF} = \frac{1}{4}(1 \pm R)(1 \pm S)$$

Avoid mixture of materials

SFP: SF with penalization



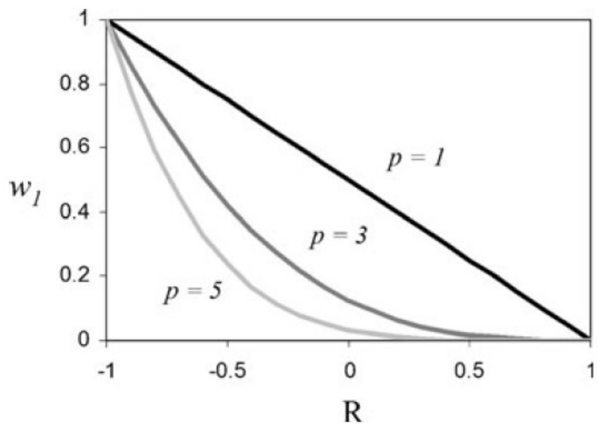
$$w_i^{SFP} = \left[\frac{1}{4}(1 \pm R)(1 \pm S)\right]^p$$

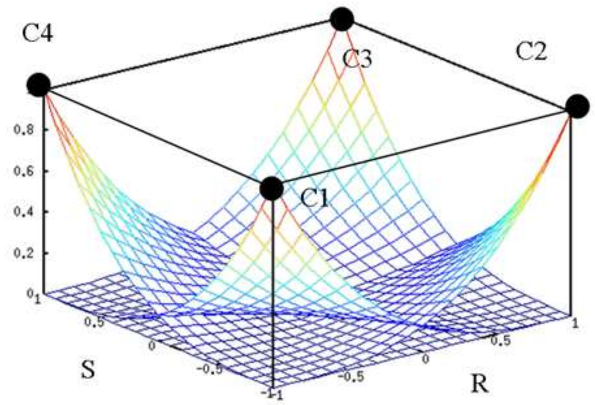
Material parameterization

- Key issue: definition of the weighting functions w_i in $\mathbf{C}^k = \sum_{i=1}^{n^k} w_i^k \mathbf{C}_i^k$
- In the literature:
  - SFP (Shape Function Parameterization) by Bruyneel (2011)
    - Here for 4 candidate materials, but possible extension to more (or less) materials
    - 2 design variables for 4 candidate materials

SFP: SF with penalization

Similar to topology optimization (SIMP approach: $E=\mu^p E_0$ and $\rho=\mu\rho_0$)





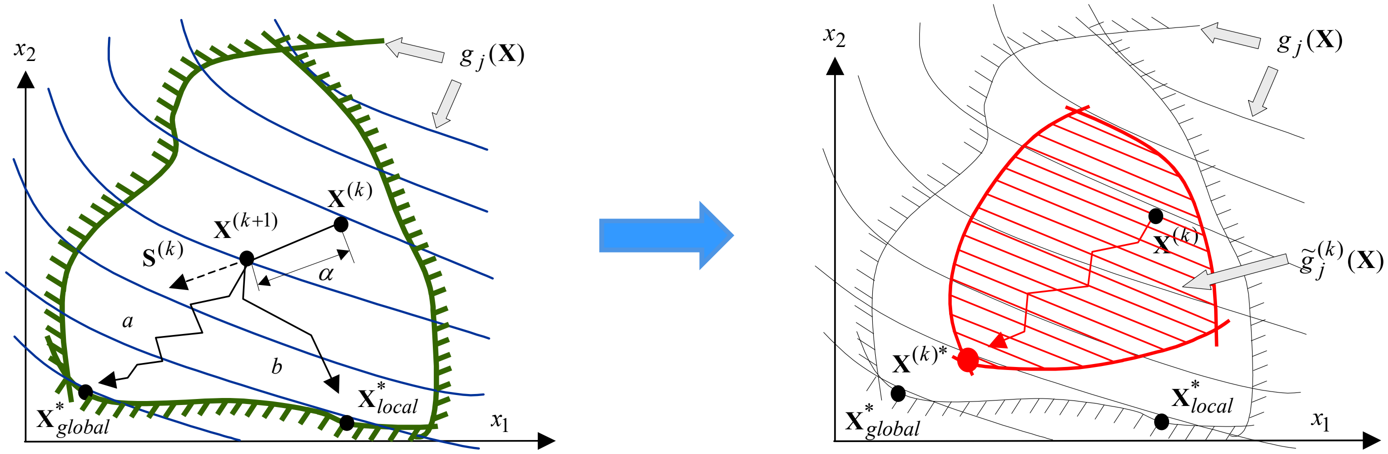
$$w_i^{SFP} = \left[\frac{1}{4}(1 \pm R)(1 \pm S)\right]^p$$

Outline of the presentation

- Problem statement
- Material parameterization
- Optimization algorithm
- Design rules constraints
- Ply continuity constraint
- Generalization
- Conclusions

Optimizer for continuous design variables

– Sequential Convex Programming approach (SCP)
  - Approximation concept approach
  - Gradient-based optimization method
  - Based on specific/tailored Taylor series expansions



$$\min g_0(\mathbf{X}) \qquad \begin{array}{l} g_j(\mathbf{X}) \le \underline{g}_j^{\max} \\ \underline{x}_i \le x_i \le \overline{x}_i \end{array}$$

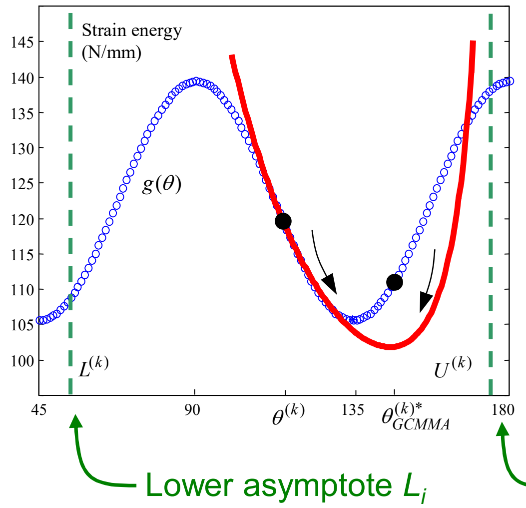
$$\min \tilde{g}_0^{(k)}(\mathbf{X}) \qquad \begin{array}{l} \tilde{g}_j^{(k)}(\mathbf{X}) \le g_j^{\max} \\ \underline{x}_i^{(k)} \le x_i \le \overline{x}_i^{(k)} \end{array}$$

Optimizer for continuous design variables

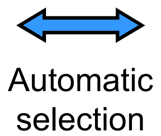
– Sequential Convex Programming approach (SCP)

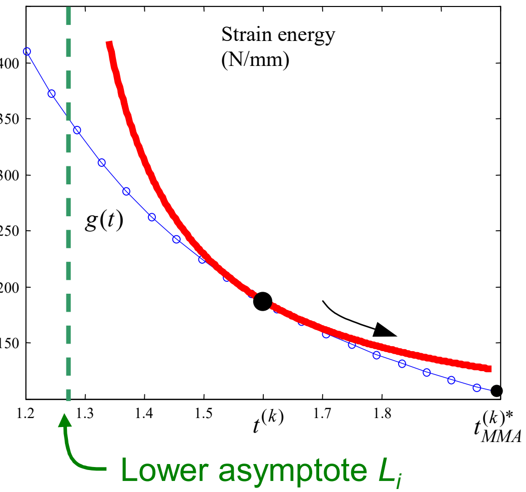
- Approximation used here: extension of Method of Moving Asymptotes (Bruyneel, Duysinx & Fleury, 2002)

$$\tilde{g}_j^{(k)}(\mathbf{x}) = g_j(\mathbf{x}^{(k)}) + \sum_{i\in A} p_{ij}^{(k)}\left(\frac{1}{U_i^{(k)}-x_i}-\frac{1}{U_i^{(k)}-x_i^{(k)}}\right) + \sum_{i\in A} q_{ij}^{(k)}\left(\frac{1}{x_i-L_i^{(k)}}-\frac{1}{x_i^{(k)}-L_i^{(k)}}\right) + \sum_{+,i\in B} p_{ij}^{(k)}\left(\frac{1}{U_i^{(k)}-x_i}-\frac{1}{U_i^{(k)}-x_i^{(k)}}\right) + \sum_{-,i\in B} q_{ij}^{(k)}\left(\frac{1}{x_i-L_i^{(k)}}-\frac{1}{x_i^{(k)}-L_i^{(k)}}\right)$$



Non monotonous approximation (group ***A***)

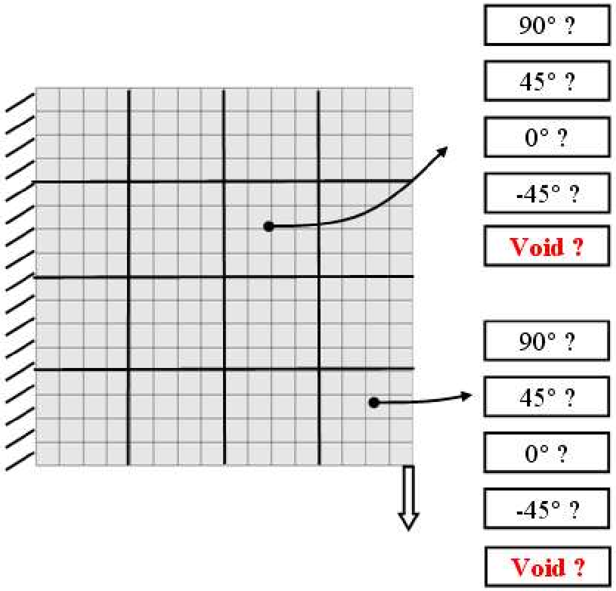




Monotonous approximation (group ***B***)

Illustration

– 16 domains



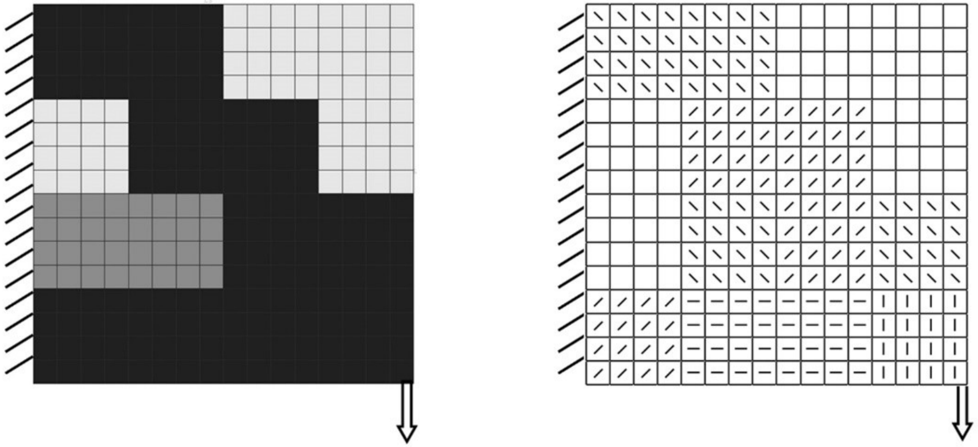
Min Compliance

$$\sum_{l=1}^{16} \mu_l \leq 11$$

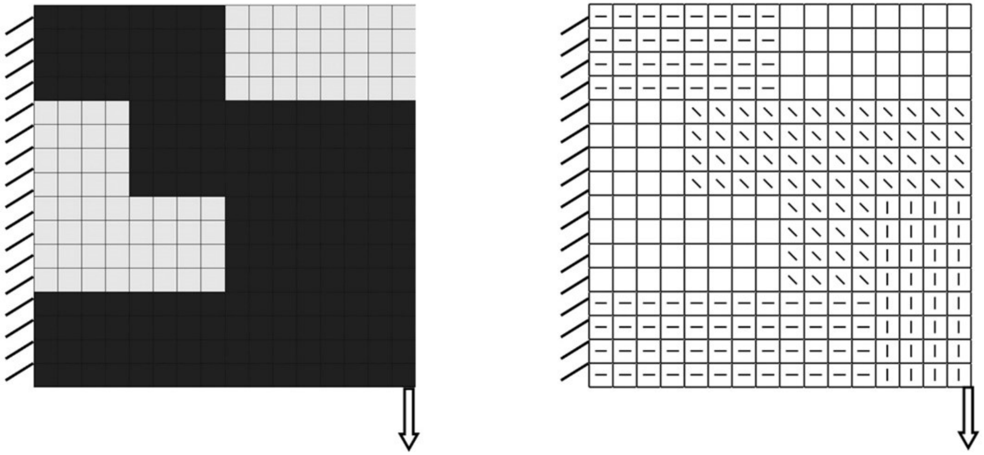
$$\mathbf{C}^l = (\mu_l)^q \left(\sum_{i=1}^{n^l} w_i^l \mathbf{C}_i^l \right)$$

μ = pseudo density for topology optimization

DMO (p=5)



SFP (p=3)



Outline of the presentation

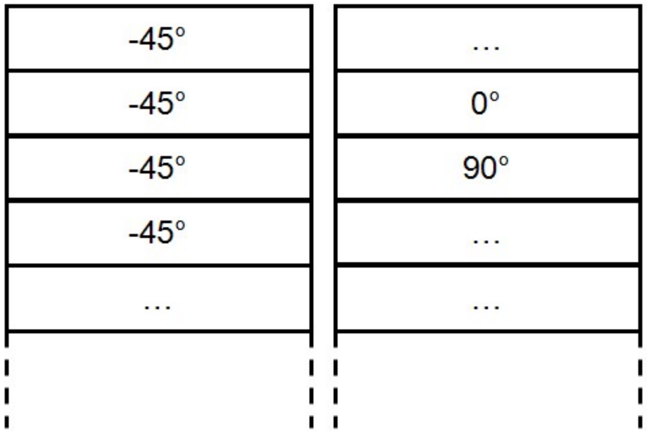
- Problem statement
- Material parameterization
- Optimization algorithm
- Design rules constraints
- Ply continuity constraint
- Generalization
- Conclusions

Design rules constraints

– Stacking sequence optimization with design rules

- Specific design rules for conventional orientations

(R1) Minimum percentage of each orientation
(R2) Balanced lay-up (same number of plies at 45° and -45°)
(R3) Symmetric laminate
(R4) No more than Nmax successive plies with the same angle
(R5) Maximum gap between two adjacent (superposed) plies is 45°



Some configurations which are not allowed
(N^{max} = 3)

Design rules constraints

– Stacking sequence optimization with design rules

- Illustration for design rule R1: Minimum percentage of each orientation

For orientations only $\underline{\xi}_j \leq \sum_{k=1}^{n} w_j^{(k)} \leq \overline{\xi}_j$ $\quad j = 1,\ldots,4$

Example for 0°: $\underline{\xi}_{0^\circ} \leq \sum_{k=1}^{n} w_{0^\circ}^{(k)} \leq \overline{\xi}_{0^\circ}$ with $\underline{\xi}_{0^\circ} = 0.1n \quad \overline{\xi}_{0^\circ} = 0.5n$

For orientations and topology optimization $\underline{\xi}_j \leq \sum_{k=1}^{n/2} \mu^{(k)} w_j^{(k)} \leq \overline{\xi}_j$ $\quad j = 1,\ldots,4$

- Illustration for design rule R2: Balanced lay-up (same number of plies at 45°/-45°)

$$\left(\sum_{k=1}^{n} w_1^{(k)} - \sum_{k=1}^{n} w_3^{(k)} \right)^2 \leq 0$$

Design rules constraints

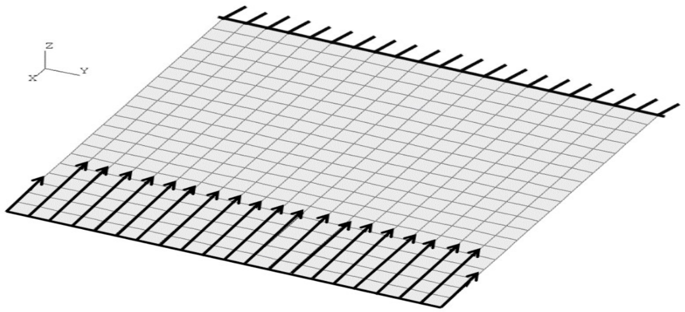
– Application 1

Maximize the first buckling load factor λ_1

Design rules taken into account

Thickness = 20 plies (symmetric)

(R1) Minimum percentage of each orientation
(R2) Balanced lay-up (same numb. of plies at 45°/-45°)
(R3) Symmetric laminate
(R4) Less than Nmax successive plies with the same angle
(R5) Maximum gap between two adjacent plies is 45°



Design rules	Iterations	Relative λ_1	Stacking sequence
R3	4	1.00	$[0_{10}]_s$
R3,R4	18	0.82	$[0_2/90/0_3/90/0_2/90]_s$
R1,R2,R3,R4	40	0.78	$[0_3/90/0/90/-45/45_2/-45]_s$
R1,R2,R3,R4,R5	27	0.72	$[0_2/-45_2/0_2/(45/90)_2]_S$

Design rules constraints

– Application 2

Minimize the compliance

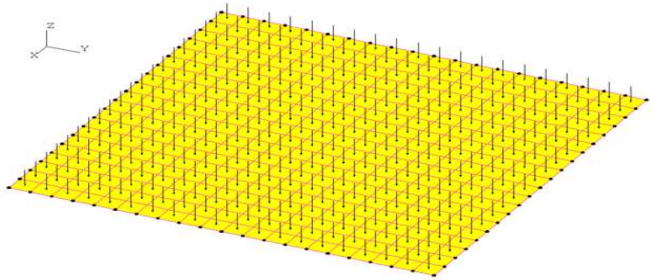
Design rules taken into account

Thickness = 20 plies (symmetric)

Remove 4 plies : use of topology design variables for each ply ($\mu^{(k)}$)

(R1) Minimum percentage of each orientation
(R2) Balanced lay-up (same numb. of plies at 45°/-45°)
(R3) Symmetric laminate
(R4) Less than Nmax successive plies with the same angle
(R5) Maximum gap between two adjacent plies is 45°

Uniform pressure



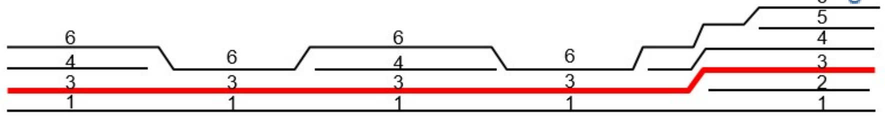
Design rules taken into account	Resulting stacking sequences	Number of iterations
R1, R3	$(45_4/-45_2/0/90/\_/\_)_s$	45
R2, R3	$(45_4/-45_4/\_/\_)_s$	23
R1, R2, R3	$(45_3/-45_2/90/0/-45/\_/\_)_s$	32
R1, R3, R4	$(45_2/-45/45/-45/0_2/90/\_/\_)_s$	30
R3, R4, R5	$(45_2/90_2/45_3/90/\_/\_)_s$	33
R1, R2, R3, R4	$(45_2/90/45/-45_2/0/-45/\_/\_)_s$	28
R2, R3, R4, R5	$(45_3/90/-45_2/0/-45/\_/\_)_s$	43
R1, R2, R3, R4, R5	$(45_3/0/-45_3/90/\_/\_)_s$	32

Outline of the presentation

- Problem statement
- Material parameterization
- Optimization algorithm
- Design rules constraints
- Ply continuity constraint
- Generalization
- Conclusions

Ply continuity constraint

– Variable thickness optimization



Specific parameterization (for the ply-drops)

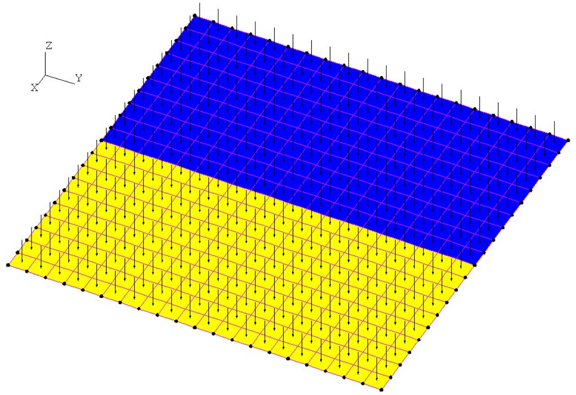
Maximize the stiffness (bending)

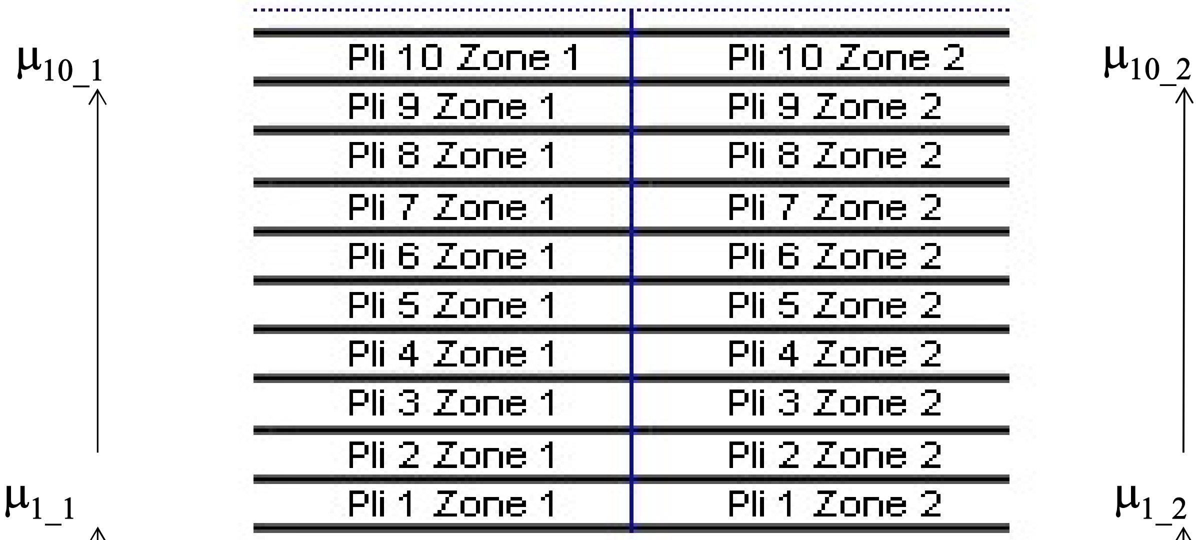
Design rules taken into account

Thickness = 20 plies

Keep 16 plies in region 1

Keep 14 plies in region 2





Presence or absence of a ply at the solution, in region 1

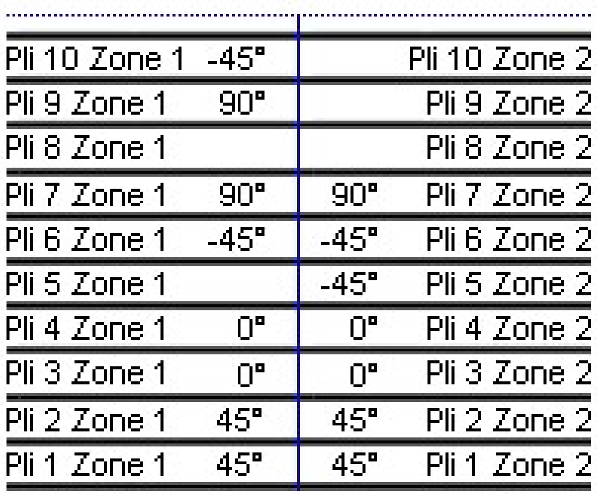
Presence or absence of a ply at the solution, in region 2

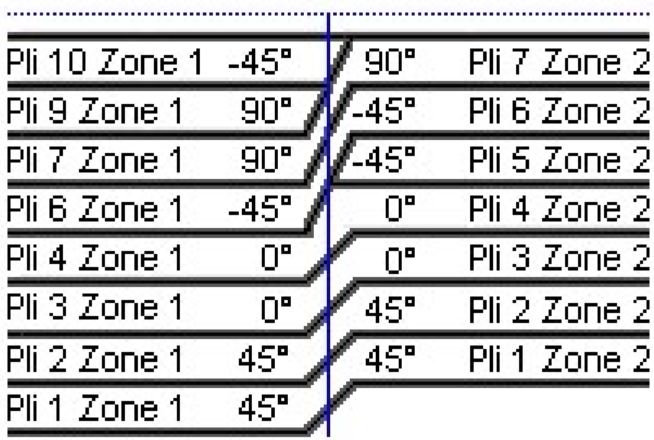
Ply continuity constraint

– Variable thickness optimization

- Solution found

Ply 8 removed from the two regions
Ply 5 removed from region 1
Plies 9 and 10 removed from region 2





Ply continuity constraint

GDTECH engineering

– Variable thickness optimization

- Solution found

(R1) Minimum percentage of each orientation
(R2) Balanced lay-up (same number of plies at 45° and -45°)
(R3) Symmetric laminate
(R4) No more than Nmax successive plies with the same angle
(R5) Maximum gap between two adjacent (superposed) plies is 45°

Design rules taken into account	Resulting stacking sequences in zone 1	Resulting stacking sequences in zone 2
R1, R3	$(45_2/0/45/-45_2/90/-45/\_/\_)_s$	$(45_2/0/45/-45_2/90/\_/\_/\_)_s$
R2, R3	$(45_4/\_/\_/-45_4)_s$	$(45_3/\_/-45/0/-45_2/\_/\_)_s$
R5, R3	$(45_3/0_2/-45_2/0/\_/\_)_s$	$(45_3/0_2/-45_2/\_/\_/\_)_s$
R1, R3, R4	$(45_2/\_/45/-45/0/-45/0/\_/90)_s$	$(45_2/90/45/-45/0/-45/\_/\_/\_)_s$
R1, R2, R3, R4, R5	$(45_2/0_2/\_/-45/90/\_/90/-45)_s$	$(45_2/0_2/-45_2/90/\_/\_/\_)_s$

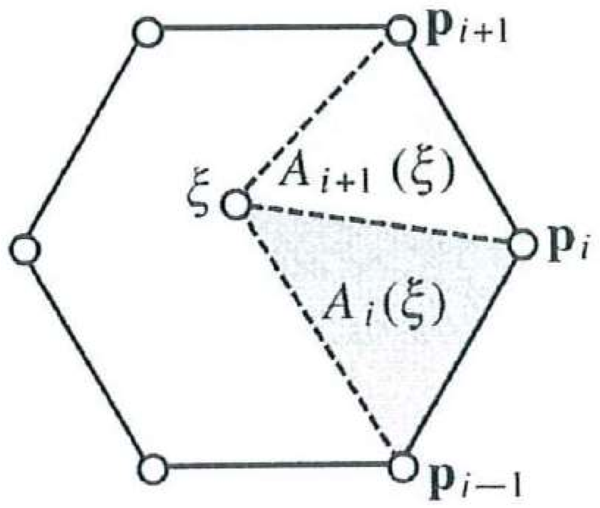
Outline of the presentation

- Problem statement
- Material parameterization
- Optimization algorithm
- Design rules constraints
- Ply continuity constraint
- Generalization
- Conclusions

Generalization

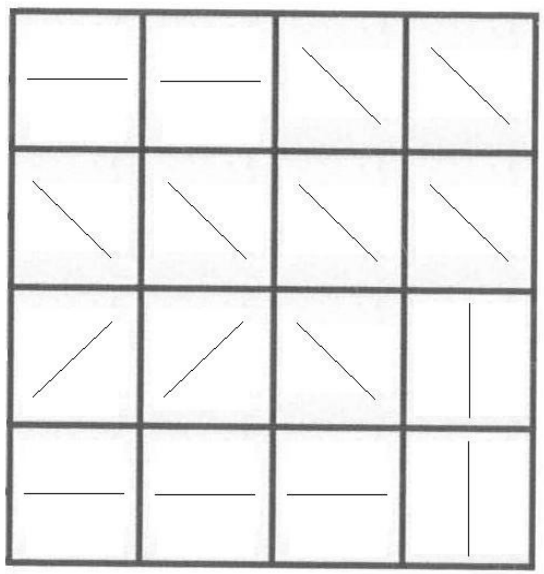
- Extension to n candidate materials and sublaminates
  - GSFP = generalized SFP
    - Use of the Washpress shape function (Polygonal shape functions)
    - 2 design variables for n candidate materials

$$w_i^{\mathrm{GSFP}} = \left(\frac{\alpha_i(\xi)}{\sum_{j=1}^{n} \alpha_j(\xi)} \right)^p \quad \text{with} \quad \alpha_i(\xi) = \frac{1}{A_i(\xi) A_{i+1}(\xi)}$$

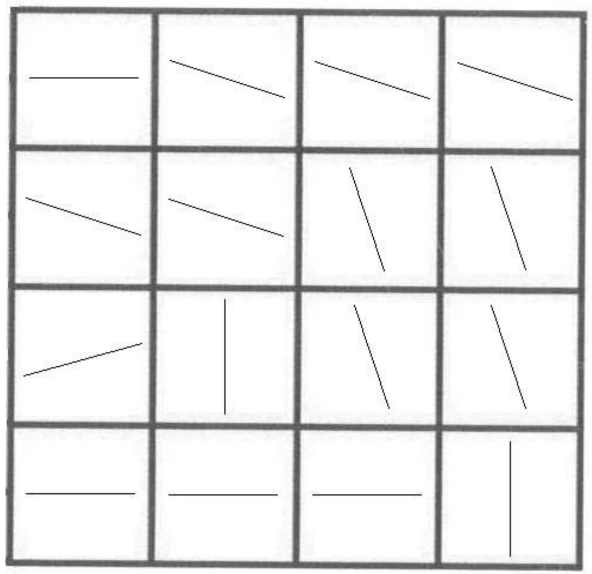


Generalization

– Extension to n candidate materials



SFP (p=3)
0°; 90°; -45°; 45°



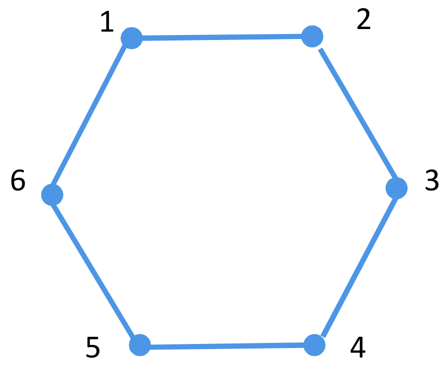
GSFP (p=3)
0°; 60°; 90°;-60°; -30°

Generalization

– Extension to n candidate materials and sublaminates

- GSFP = Generalized SFP
- Application to sublaminates
  - Instead of distributing orientations in each ply, sub-laminates (specified sets of plies) can be distributed in the structure

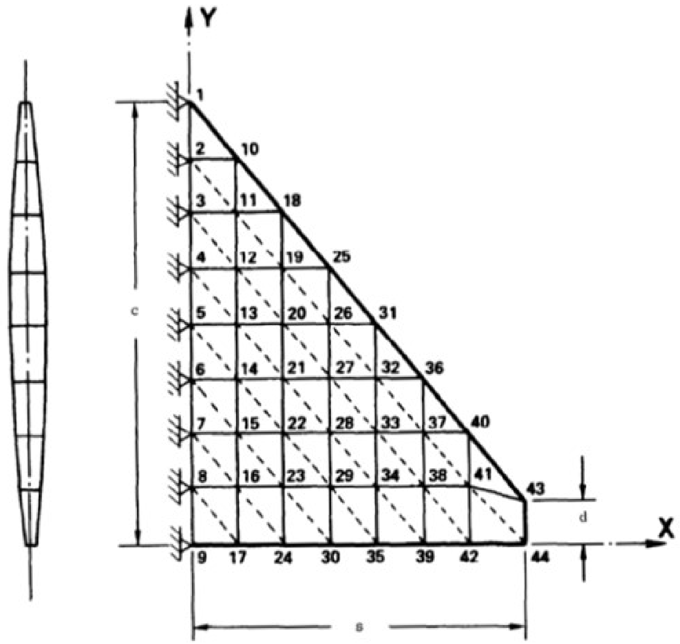
Sub-laminate number	Candidate sub-laminates
1	[±45/0_2/90/0_2/90/0_2]
2	[±45/0/90/0/90/0]
3	[±45/0/90/0]
4	[±45/0/90]
5	[±45/90/0/90/0/90]
6	[±45/90_2/0/90_2/0/90_2]

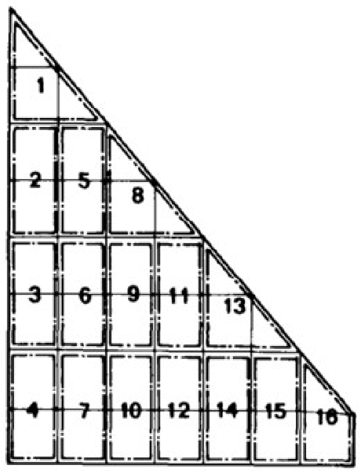


Generalization

GDTECH engineering

– Extension to sublaminates
  - Illustration on the Delta wing:
    - 16 regions
    - Selection of the optimal laminate in each region





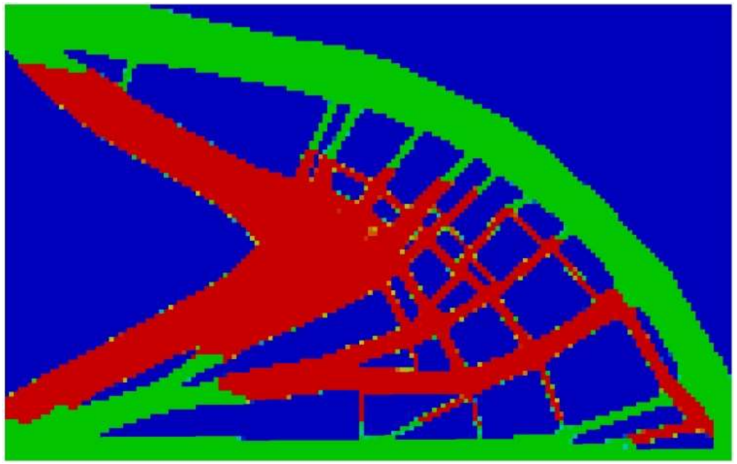
Region of the Delta wing	Optimal sub-laminate
1	$[\pm45/0_2/90/0_2/90/0_2]$
2	[±45/0/90/0/90/0]
3	[±45/0/90/0/90/0]
4	[±45/0/90/0/90/0]
5	$[\pm45/90_2/0/90_2/0/90_2]$
6	[±45/0/90/0/90/0]
7	[±45/0/90/0/90/0]
8	[±45/90/0/90/0/90]
9	[±45/0/90/0/90/0]
10	[±45/0/90/0/90/0]
11	[±45/0/90/0]
12	[±45/0/90/0/90/0]
13	[±45/0/90]
14	[±45/0/90/0/90/0]
15	[±45/0/90/0/90/0]
16	[±45/0/90]

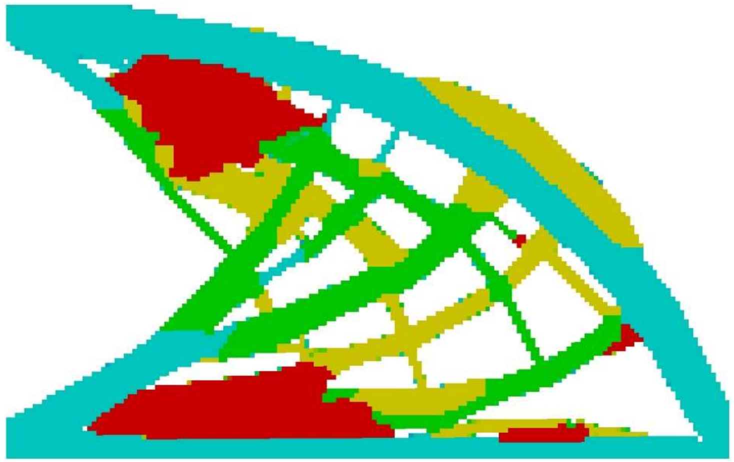
Generalization

– Extension to multi-material topology optimization

2 materials + void
Min Compliance
Constraint on the total mass
$E_2 = 57\%\ E_1$
$\rho_2 = 50\%\ \rho_2$

Material 1 : E = 210 E9 ; V_{max} = 20 %
Material 2 : E = 150 E9 ; V_{max} = 10 %
Material 3 : E = 90 E9 ; V_{max} = 10 %
Material 4 : E = 40 E9 ; V_{max} = 10 %





Outline of the presentation

GDTECH engineering

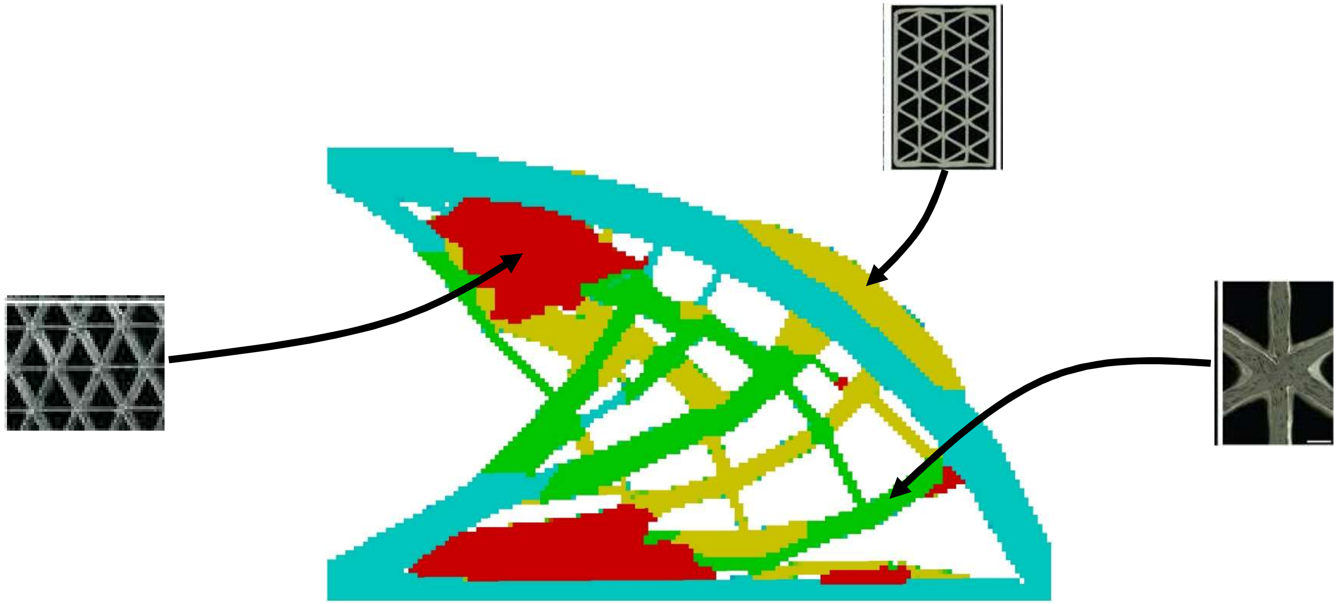
- Problem statement
- Material parameterization
- Optimization algorithm
- Design rules constraints
- Ply continuity constraint
- Generalization
- Conclusions

Conclusions

– Composite structure optimization = difficult task
– It is demonstrated that it's possible to solve the problem with continuous design variables
    - Discrete => continuous thanks to a specific parameterization
    - For conventional orientations (0°, 45°, -45°, 90°)
    - Taking into account the design rules
    - Taking into account the ply continuity constraint
    - Generalization to n candidate materials/orientations
– Application to multi-material topology optimization

Conclusions

– Next step: application to lattice structures
  - different patterns = different materials to distribute
  - Application to Additive Manufacturing



Thank you for your attention
Any question?

GDTECH engineering

Michael.bruyneel@gdtech.eu